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Constrained single period problem under demand uncertainty

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Abstract In this paper, we develop the multi-product, multi-constraint, Single Period Problem (SPP) with uncertain demands, considering an incremental discount situation. Three new models are presented for multi-product, multi-constraint SPP in fuzzy, stochastic and rough environments. We consider constraints, such as service rate, restriction on order quantity and restrictions on warehouse space and budget. We also consider that the order quantity is a multiplier of predefined batch size. Furthermore, three kinds of solution algorithm, (1) harmony search, (2) hybrid intelligent based on harmony search and fuzzy simulation and (3) hybrid intelligent based on harmony search and rough simulation, are presented for the developed models to maximize expected profit. Finally, illustrative examples are presented to show the performance of the developed models and algorithms.

1. Introduction

Classic SPP is famous as a stochastic inventory control models [1]. In this problem the order should be placed at the beginning of the period based on demand estimation. If the order is more than the actual demand of the period, the company will face the holding cost of remaining items at the end of the period; otherwise he will lose the sales at the end of the period. The objective of this model is to maximize the expected profit through deciding the order quantity at the beginning of the period. In the real world, many products have a limited selling period, so the SPP model is often used to aid decision-making in fashion, sporting, service industries, etc.

Regarding a real-world application of this problem, we will discuss the fresh food business [1]. Many types of dairy products, such as cheese, milk, ice cream and yogurt, are cases for which a single period problem can be used. Many kinds of retailer, wholesaler and restaurant order dairy products daily. Similarly, a chain of very large restaurants offers their lunch-time customers a hundred different fresh "dim sum" items every day (Chinese delicacies served in small portions, somewhat similar to, but considerably more elaborate than, Spanish tapas) [1]. In the above-mentioned situation, there may exist several limitations, such as capacity, maximum order quantity, batch size, budget or space constraints. Therefore, the problem is to determine the optimal order quantity of each product at the beginning of each work-period, which may be a day, a week or even a month. In the above mentioned markets, the demand rate of products usually follows an uncertain pattern.

In some cases it may be possible to apply a stochastic distribution for demand behavior, while in many cases there is no appropriate stochastic pattern for demand of these products. Thus, considering fuzzy or rough theories in the approximate estimation of products' demand may be useful [1].

Obviously, the creation of practical uncertain models in inventory control systems is important, because an uncertain environment is a fact that each system may face. According to the above explanation, we introduce three models, stochastic, fuzzy theory and rough theory, to estimate demands. We name the stochastic multi-product, multi-constraint, single period problem: "SSPP", and similarly, for fuzzy and rough models: "FSPP" and "RSPP".

The motivation behind considering these three kinds of uncertainty in this research is provision of a solution for managers in cases of uncertainty for demand. If they have good

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and sufficient historical data, then, they can use the stochastic model (SSPP). But, if they have no information or knowledge concerning the demand, they can consider the rough model (RSPP). In cases where there is some information, we can estimate demand approximately, by utilizing the fuzzy set theory, and then, the fuzzy model (FSPP) can be used. To the best of our knowledge, the proposed RSPP model is a new model in this area.

Since the introduced models are different and it is not possible to use a specific solution approach to solve them, we develop three different solution approaches based on Meta-heuristic algorithm (Harmony Search) and Simulation (Fuzzy and Rough).

2. Literature review

Classical SPP and its various extensions are widely studied by Khouja [2]. For extensions about different objectives and utility functions, Lau [3], Anvari [4] and Lau and Lau [5] proposed models of SPP in which the objectives were to maximize the probability of achieving a target profit. Atkinson [6], Anvari and Kusy [7] and Chung [8] used different effectiveness and risk tolerance criteria. In extensions about different discount policies, Anvari [4], Pfeifer [9], Anvari and Kusy [7], Chung [8] and Khouja [10] formulated different sale discounts for different sale quantities. For extensions about multi-product, multi-constraint, Lau and Lau [11], Lau and Lau [12], Abdel-Malek and Montanari [13] considered a multi-product problem with budget constraints, and Taleizadeh et al. [14] developed a new model of multi-product, multi-constraint SPP. Matsuyama [15] and Alfares and Elmorra [16] developed SPP for a single product, in two- and multi-period states, respectively. Lushu et al. [17] developed two models for fuzzy SPP for the first time. The first was SPP with probabilistic demand and fuzzy cost components, while in the second, the demand rate follows a fuzzy pattern, and cost factors were deterministic. For other research on fuzzy SPP, we can refer to Chiang and Wen-Kai [18], who developed a SPP model in which demand was assumed fuzzy. Dutta et al. [19] presented a SPP model in a mixed imprecise and uncertain environment in which demand rate was assumed a fuzzy random variable. The previous studies were developed by Ji and Shao [20] who considered quantity discount in fuzzy SPP. Then, Shao and Ji [21] added a budget constraint in their previous research [20] and developed a new constrained fuzzy SPP model. Taleizadeh et al. [22] developed a newsboy model, in a multi-product, multi-constraint situation, with fuzzy demand and incremental discount. In this research, we developed the SPP problem for cases of multi-products, multi-constraints, considering uncertain demands. To develop the new models, we considered incremental discount, purchasing, holding, transportation and shortage costs, and batch order size.

We can summarize the main differences between our models and previous models as: (1) Lost sale cost is noticed (2) Incremental discount is used (3) Multi constraints are considered (4) Order quantity can be a multiplier of a pre-defined batch size (5) Uncertain demands (fuzzy, stochastic and rough) are noticed and (6) Transportation cost is also included in the model.

3. Problem definition and formulation

Consider an inventory problem in which an order should be undertaken just at the beginning of the period and the demand rate can follow fuzzy, stochastic and rough patterns. The constraints of the inventory system are batch ordering policy, warehouse and budget restrictions and service level. Unsatisfied demand will be lost and an incremental discount will be offered from the company. Also the costs of objective function including shortage, holding and transportation are assumed to be deterministic and they will be deployed at the end of the period.

Considering zero lead time is one of the main assumptions in the Newsvendor problem. Moreover in many applied cases (for example fresh food business), there is a good and efficient distribution system and the lead time is very short. Therefore considering the value of zero for lead time can be reasonable in many applied cases which we also assumed.

In the following sub-sections we will discuss parameter definitions and problem modeling.

3.1. Problem parameters and variables

In this paper the following notations, which some of them are used from [1,14,22], are:

T: Number of products ($J = 1, 2, \dots, T$).

Q_j: Decision variable representing the order quantity of product $j (Q_j = n_j m_j)$.

m_j: Decision variable representing the batch order quantity of product j.

n_j: Number of goods in each batch of product j (Batch Size).

$\hat{\pi}_j$: The shortage cost per unit demand of product j at the end of period.

FI_j: The fraction of purchasing cost to calculate holding cost of product j.

h_j: The holding cost per unit inventory of product j at the end of the period, $h_j = FI_j * C_j$.

p_j: The sales price per unit of product j.

$\tilde{\theta}_j$: The fuzzy demand of product j.

D_j: The stochastic demand of product j following Poisson distribution.

λ_j: The expected value of $D_j \lambda_j = E[D_j]$.

δ_j: The rough demand of product j.

SL_j: The lower limit of the service rate for the product j.

UL_j: The upper limit of the order quantity for the product j.

f_j: The required storage space per unit of product j.

F: Total available warehouse space.

$\hat{f}$: Total available warehouse space in each shipment

W: Total available budget.

q_{ij}: The ith discount point of the product $j (i : 1, 2, \dots, n)$.

c_{ij}: The purchasing cost per unit of product j at the ith discount point.

Q_{ij}: The order quantity of the product j at the ith discount price.

$C(Q_j)$: The expected purchasing cost of product j in the period.

$H(Q_j)$: The expected holding cost of product j in the period.

$S(Q_j)$: The expected shortage cost of product j in the period.

$T(Q)$: The total transportation cost in each period.

$C_T(Q_j)$: The expected total cost of product j in the period.

$R(Q_j)$: The expected revenue of product j in the period.

$Z(Q)$: The total expected profit in the period.

3.2. FSPP with incremental discount policy

In this section the demands are assumed to be fuzzy. The orders can take place at once and at the beginning of the period and no opportunity is allowed for replenishing the stock. The transportation cost $T(Q)$ is formulated as below:

$$T(Q)=\begin{cases} A; & 0<\sum_{j=1}^{T} f_j m_j \le \hat{f} \\ 2A; & \hat{f}<\sum_{j=1}^{T} f_j m_j \le 2\hat{f} \\ \vdots & \vdots \\ mA; & (m-1)\hat{f}<\sum_{j=1}^{T} f_j m_j \le m\hat{f}. \end{cases} \tag{1}$$

By introducing the binary variables Y_k; $k=1,2,\ldots,m$, the transportation cost will be:

$$\begin{aligned} &T(Q)=\sum_{k=1}^{m} kAY_k \\ &0<\sum_{j=1}^{T} f_j m_j \le \hat{f}Y_1 \\ &\hat{f}Y_2<\sum_{j=1}^{T} f_j m_j \le 2\hat{f}Y_2 \\ &\vdots \\ &(m-1)\hat{f}Y_m<\sum_{j=1}^{T} f_j m_j \le m\hat{f}Y_m \\ &Y_1+Y_2+\cdots+Y_m=1, \qquad Y_k=0,1 \quad \forall k=1,2,\ldots,m. \end{aligned} \tag{2}$$

If the total demand is equal to or more than order quantity, the revenue is P_jQ_j, and the shortage cost as lost sales is $\hat{\pi}_j(\tilde{\theta}_j-Q_j)$. If the total demand is less than the order quantity, the revenue is $P_j\tilde{\theta}_j$, and the holding cost of remained inventory is $h_j(Q_j-\tilde{\theta}_j)$. So the total expected profit function can be written as:

$$\begin{aligned} Z(Q) &= \sum_{j=1}^{T}\left[R(Q_j)-C_T(Q_j)\right] \\ &= \begin{cases} \sum_{j=1}^{T}\left[P_j\tilde{\theta}_j-C(Q_j)-h_j(Q_j-\tilde{\theta}_j)\right]-\sum_{k=1}^{m}kAY_k \\ \quad \tilde{\theta}_j \le Q_j \\ \sum_{j=1}^{T}\left[P_jQ_j-C(Q_j)-\hat{\pi}_j(\tilde{\theta}_j-Q_j)\right]-\sum_{k=1}^{m}kAY_k \\ \quad \tilde{\theta}_j \ge Q_j. \end{cases} \end{aligned} \tag{3}$$

The purchasing cost of the company for the jth product at the beginning of a period can be calculated using the incremental discount policy. Let the incremental discount policy be

$$C(Q_j)=\begin{cases} C_{1j}Q_j; \quad 0<Q_j\le q_{1j} \\ C_{1j}q_{1j}+C_{2j}(Q_j-q_{2j}); \quad q_{1j}<Q_j\le q_{2j} \\ \vdots \\ C_{1j}q_{1j}+C_{2j}(q_{2j}-q_{1j})+\cdots+C_{nj}(Q_j-q_{n-1,j}); \\ \quad Q_j\ge q_{nj}, \end{cases} \tag{4}$$

where q_{ij} and C_{ij}; $i=1,2,\ldots,n$ are the discount points and purchasing costs for each unit of the jth product that correspond to its lth discount break point, respectively. In order to include the incremental discount policy in the inventory model, we use Eq. (5) to model the incremental discount policy:

$$\begin{aligned} &C(Q_j)=C_{1j}Q_{1j}+C_{2j}Q_{2j}+\cdots+C_{nj}Q_{nj} \\ &Q_j=Q_{1j}+Q_{2j}+\cdots+Q_{nj} \\ &q_{1j}\lambda_{2j}\le Q_{1j}\le q_{1j}\lambda_{1j} \\ &\left(q_{2j}-q_{1j}\right)\lambda_{3j}\le Q_{2j}\le\left(q_{2j}-q_{1j}\right)\lambda_{2j} \\ &\vdots \\ &0\le Q_{nj}\le M\lambda_{nj} \\ &\lambda_{1j}\ge\lambda_{2j}\ge\cdots\ge\lambda_{nj}, \qquad \lambda_{ij}=0,1 \quad \forall i,\ i=1,2,\ldots,n. \end{aligned} \tag{5}$$

M is a very big number.

By this modeling, and another binary variable such as X_j, the FSPP with the incremental discount policy becomes:

$$\begin{aligned} Z(\tilde{\xi},Q) = &\sum_{j=1}^{T}\left[P_j\tilde{\theta}_j-h_j(Q_j-\tilde{\theta}_j)\right]X_j \\ &+\sum_{j=1}^{T}\left[P_jQ_j-\hat{\pi}_j(\tilde{\theta}_j-Q_j)\right](1-X_j) \\ &-\sum_{j=1}^{T}\sum_{i=1}^{n}C_{ij}Q_{ij}-\sum_{k=1}^{m}kAY_k \end{aligned} \tag{6}$$

$$\text{S.t.}: \sum_{j=1}^{T} f_j\cdot m_j\le F \tag{7}$$

$$\sum_{j=1}^{T}\sum_{i=1}^{n}C_{ij}Q_{ij}\le W \tag{8}$$

$$Q_j\le UL_j \quad \forall j, j=1,2,\ldots,T \tag{9}$$

$$Q_j=n_jm_j \quad \forall j, j=1,2,\ldots,T \tag{10}$$

$$SL_j\left[\frac{1}{4}(\theta_{1j}+2\theta_{2j}+\theta_{3j})\right]\le Q_j \quad \forall j, j=1,2,\ldots,T \tag{11}$$

$$\begin{cases} Q_j=Q_{1j}+Q_{2j}+\cdots+Q_{nj} \\ q_{1j}\lambda_{2j}\le Q_{1j}\le q_{1j}\lambda_{1j} \\ \left(q_{2j}-q_{1j}\right)\lambda_{3j}\le Q_{2j}\le\left(q_{2j}-q_{1j}\right)\lambda_{2j} \\ \vdots \\ 0\le Q_{nj}\le M\lambda_{nj} \\ \lambda_{1j}\ge\lambda_{2j}\ge\cdots\ge\lambda_{nj} \\ \lambda_{ij}=0,1; \quad \forall j, j=1,2,\ldots,T, \forall i, i=1,2,\ldots,m \end{cases} \tag{12}$$

$$\begin{cases} 0<\sum_{j=1}^{P} f_j m_j\le\hat{f}Y_1 \\ \hat{f}Y_2<\sum_{j=1}^{P} f_j m_j\le 2\hat{f}Y_2 \\ \vdots \\ (m-1)\hat{f}Y_m<\sum_{j=1}^{P} f_j m_j\le m\hat{f}Y_m \\ Y_1+Y_2+\cdots+Y_m=1 \\ Y_k=0,1 \quad \forall k=1,2,\ldots,m \end{cases} \tag{13}$$

$$X_j=0,1 \quad \forall j, j=1,2,\ldots,T \tag{14}$$

$$Q_j, m_j\ge 0, \text{ integer} \quad \forall j, j=1,2,\ldots,T. \tag{15}$$

The first constraint, (7) presents the warehouse capacity, where f_j is the required space per unit of the product j and F is the total available space in warehouse. The second constraint,

(8) is the budget constraint and the third constraint, (9) states that the order quantity cannot be more than the upper limit. The forth constraint, (10), which presents the order quantity can be a multiplier of a batch size, and finally the constraint (11) presents the service rate constraint. Service rate is modeled based on the SPP policy. We assume that the service rate is satisfying the percent SL_j of product j demands, or in other words the probability of facing to shortage, shall be equal to or less than $1 - SL_j$, which is formulated as constraint $\frac{E[\tilde{\theta}_j - Q_j]}{E[\tilde{\xi}]} \leq 1 - SL_j$. According to Appendix A (Definition A.3), the fuzzy service rate constraint is transformed to $E[\tilde{\theta}_j] - E[\tilde{\theta}_j](1 - SL_j) - E[Q_j] \leq 0 \Rightarrow SL_j E[\tilde{\theta}_j] \leq Q_j$ and finally, the service rate constraint based on the symmetric fuzzy variable will be $SL_j[\frac{1}{4}(\theta_{1j} + 2\theta_{2j} + \theta_{3j})] \leq Q_j$. Also equations group in (22) and (23) are resulted as constraints from modeling the discount policy and transportation costs, respectively.

3.3. SSPP with incremental discount policy

To calculate the revenue obtained from selling the jth product in a period, let us assume that if the total demand quantity is more than the order quantity, then the sold quantity is Q_j otherwise, it is X_j. In other words:

$$\text{Sold Quantity of } j\text{th Product} = \begin{cases} Q_j & \text{if } X_j \geq Q_j \\ X_j & \text{if } X_j < Q_j. \end{cases} \tag{16}$$

Since the probability mass function of demand for product j is $f_{X_j}(x_j)$, the expected sold quantity of the jth product at the end of the period is determined as $\sum_{X_j=0}^{Q_j-1} X_j \cdot f_{X_j}(x_j) + \sum_{X_j=Q_j}^{+\infty} Q_j \cdot f_{X_j}(x_j)$, Hence, the expected revenue is obtained by:

$$R(Q_j) = \sum_{X_j=0}^{Q_j-1} P_j \cdot X_j \cdot f_{X_j}(x_j) + \sum_{X_j=Q_j}^{+\infty} P_j \cdot Q_j \cdot f_{X_j}(x_j). \tag{17}$$

According to the description in 3.3 the expected value of holding and shortage costs will be as below:

$$H(Q_j) = \sum_{X_j=0}^{Q_j} h_j(Q_j - X_j) f_{X_j}(x_j) \tag{18}$$

$$S(Q_j) = \sum_{X_j=Q_j+1}^{+\infty} \hat{\pi}_j(X_j - Q_j) f_{X_j}(x_j). \tag{19}$$

For the service rate constraint, we are interested to ensure that the ratio of average shortage to the average demand of a period should be equal to or less than $1 - SL_j$. So to formulate the service rate constraint, consider the average shortage at the end of a period:

$$\sum_{X_j=Q_j+1}^{+\infty} (X_j - Q_j) f_{X_j}(x_j). \tag{20}$$

By dividing (20), to the average demand of a period we will have the service rate constraint as:

$$\frac{\sum_{X_j=Q_j+1}^{+\infty} (X_j - Q_j) f_{X_j}(x_j)}{\lambda_j} \leq 1 - SL_j. \tag{21}$$

The purchasing costs under incremental discount, transportation costs and other constraints are formulated in the same manner as the fuzzy section. So the model of SSPP will be:

$$\begin{aligned} Z(X, Q) = & \sum_{X_j=0}^{Q_j-1} P_j \cdot X_j \cdot \frac{e^{-\theta_j}\theta_j^{X_j}}{X_j!} + \sum_{X_j=Q_j}^{+\infty} P_j \cdot Q_j \cdot \frac{e^{-\theta_j}\theta_j^{X_j}}{X_j!} \\ & - \sum_{X_j=0}^{Q_j-1} \left(h_{1j}(Q_j - X_j)\right) \frac{e^{-\theta_j}\theta_j^{X_j}}{X_j!} \\ & - \sum_{X_j=Q_j+1}^{+\infty} \left(\pi_{1j}(X_j - Q_j)\right) \frac{e^{-\theta_j}\theta_j^{X_j}}{X_j!} \\ & - \sum_{j=1}^{T}\sum_{i=1}^{n} C_{ij} Q_{ij} - \sum_{k=1}^{m} kAY_k \end{aligned}$$

$$\begin{aligned} \text{S.t.}: & \sum_{j=1}^{T} f_j m_j \leq F \\ & \sum_{j=1}^{T}\sum_{i=1}^{n} C_{ij} Q_{ij} \leq W \\ & Q_j \leq UL_j \quad \forall j, j = 1, 2, \ldots, T \\ & Q_j = n_j m_j \quad \forall j, j = 1, 2, \ldots, T \\ & \frac{\sum_{X_j=Q_j+1}^{+\infty} (X_j - Q_j) \frac{e^{-\theta_j}\theta_j^{X_j}}{X_j!}}{\lambda_j} \leq 1 - SL_j \quad \forall j, j = 1, 2, \ldots, T \\ & \begin{cases} Q_j = Q_{1j} + Q_{2j} + \cdots + Q_{nj} \\ q_{1j}\lambda_{2j} \leq Q_{1j} \leq q_{1j}\lambda_{1j} \\ \left(q_{2j} - q_{1j}\right)\lambda_{3j} \leq Q_{2j} \leq \left(q_{2j} - q_{1j}\right)\lambda_{2j} \\ \vdots \\ 0 \leq Q_{nj} \leq M\lambda_{nj} \\ \lambda_{1j} \geq \lambda_{2j} \geq \cdots \geq \lambda_{nj} \\ \lambda_{ij} = 0, 1; \quad \forall j, j = 1, 2, \ldots, T, \forall i, i = 1, 2, \ldots, m \end{cases} \\ & \begin{cases} 0 < \sum_{j=1}^{P} f_j m_j \leq \hat{f} Y_1 \\ \hat{f} Y_2 < \sum_{j=1}^{P} f_j m_j \leq 2\hat{f} Y_2 \\ \vdots \\ (m-1)\hat{f} Y_m < \sum_{j=1}^{P} f_j m_j \leq m\hat{f} Y_m \\ Y_1 + Y_2 + \cdots + Y_m = 1 \\ Y_k = 0, 1 \quad \forall k = 1, 2, \ldots, m, \end{cases} \\ & X_j = 0, 1 \quad \forall j, j = 1, 2, \ldots, T \\ & Q_j, m_j \geq 0, \text{ integer} \quad \forall j, j = 1, 2, \ldots, T. \end{aligned} \tag{22}$$

3.4. RSPP with incremental discount policy

In this section, we model the rough single period problem (RSPP). If the total rough demand during the period δ_j, be equal to or more than the order quantity ($\delta_j \geq Q_j$), then the inventory level at the end of the period will be zero, otherwise ($\delta_j < Q_j$), the inventory level will be positive ($Q_j - \delta_j$). So the holding cost at the end of the period for product j, $H(Q_j)$, will be $h_j(Q_j - \delta_j)$, if $\delta_j \leq Q_j$ otherwise it will be zero. In a similar manner to calculate the holding cost, the shortage cost for product j, $S(Q_j)$ will be $\hat{\pi}_j(\delta_j - Q_j)$ if $\delta_j \geq Q_j$ otherwise it will be zero. The revenue from product $j$at the end of the period, $R(Q_j)$, is $P_j\delta_j$ if $\delta_j \leq Q_j$ otherwise it will be P_jQ_j. So the total expected profit due to all products will be:

$$Z(\delta, Q) = \sum_{i=1}^{T}\left[R(Q_j) - C_T(Q_j)\right]$$
$$= \begin{cases} \sum_{i=1}^{T}\left[P_j\delta_j - c_jQ_j - h_j(Q_j - \delta_j)\right] - \sum_{k=1}^{m} kAY_k \\ \quad \delta_j \leq Q_j \\ \sum_{i=1}^{T}\left[P_jQ_j - c_jQ_j - \hat{\pi}_j(\delta_j - Q_j)\right] - \sum_{k=1}^{m} kAY_k \\ \quad \delta_j \geq Q_j. \end{cases} \tag{23}$$

Since the ratio of the expected value of the shortage quantity to the expected value of the demand quantity is $\frac{E[\delta_j - Q_j]}{E[\delta_j]}$ and according to definition (8), the service rate constraint based on an assumed rough variable ($[a, b][c, d]$) will be:

$$\frac{E[\delta_j - Q_j]}{E[\delta_j]} = \frac{E[\delta_j] - Q_j}{E[\delta_j]} = \frac{\frac{1}{4}(a_j + b_j + c_j + d_j) - Q_j}{\frac{1}{4}(a_j + b_j + c_j + d_j)} \leq 1 - SL_j. \tag{24}$$

So, we will have:

$$\frac{1}{4}(a_j + b_j + c_j + d_j)SL_j \leq Q_j. \tag{25}$$

Finally, the RSPP will be as below:

$$Z(\delta, Q) = \sum_{j=1}^{T}\left[P_j\delta_j - h_j(Q_j - \delta_j)\right]X_j + \sum_{j=1}^{T}\left[P_jQ_j - \hat{\pi}_j(\delta_j - Q_j)\right](1 - X_j) - \sum_{j=1}^{T}\sum_{i=1}^{n} C_{ij}Q_{ij} - \sum_{k=1}^{m} kAY_k$$

$$\text{S.t.}: \sum_{j=1}^{T} f_j.m_j \leq F$$
$$\sum_{j=1}^{T}\sum_{i=1}^{n} C_{ij}Q_{ij} \leq W$$
$$Q_j \leq UL_j \quad \forall j, j = 1, 2, \ldots, T$$
$$Q_j = n_jm_j \quad \forall j, j = 1, 2, \ldots, T$$
$$\frac{1}{4}(a_j + b_j + c_j + d_j)SL_j \leq Q_j \quad \forall j, j = 1, 2, \ldots, T$$
$$\begin{cases} Q_j = Q_{1j} + Q_{2j} + \cdots + Q_{nj} \\ q_{1j}\lambda_{2j} \leq Q_{1j} \leq q_{1j}\lambda_{1j} \\ \left(q_{2j} - q_{1j}\right)\lambda_{3j} \leq Q_{2j} \leq \left(q_{2j} - q_{1j}\right)\lambda_{2j} \\ \vdots \\ 0 \leq Q_{nj} \leq M\lambda_{nj} \\ \lambda_{1j} \geq \lambda_{2j} \geq \cdots \geq \lambda_{nj}; \qquad \lambda_{ij} = 0, 1; \\ \quad \forall j, j = 1, 2, \ldots, T, \forall i, i = 1, 2, \ldots, m \end{cases}$$
$$\begin{cases} 0 < \sum_{j=1}^{P} f_jm_j \leq \hat{f}Y_1 \\ \hat{f}Y_2 < \sum_{j=1}^{P} f_jm_j \leq 2\hat{f}Y_2 \\ \vdots \\ (m-1)\hat{f}Y_m < \sum_{j=1}^{P} f_jm_j \leq m\hat{f}Y_m \\ Y_1 + Y_2 + \cdots + Y_m = 1; \qquad Y_k = 0, 1 \\ \quad \forall k = 1, 2, \ldots, m \end{cases}$$
$$Q_j, m_j \geq 0, \text{integer}, \qquad X_j = 0, 1 \quad \forall j, j = 1, 2, \ldots, T. \tag{26}$$

4. A hybrid intelligent algorithm

Since the developed models are mixed integer programming or mixed integer nonlinear programming, reaching an analytical solution (if any) to the problem is difficult [23]. Therefore to solve the models under different criteria, we developed a hybrid intelligent algorithm of fuzzy simulation and harmony search for FSPP. A harmony search algorithm is developed to solve the SSPP and a hybrid intelligent algorithm including the rough simulation and harmony search algorithm was developed for RSPP.

4.1. Fuzzy simulation

In this paper we have used a fuzzy simulation technique to estimate the fuzzy demands of the FSPP model. Denoting $\tilde{\theta}_i$ by $\tilde{\theta}_i = (\tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_t)$, μ as the membership function of $\tilde{\theta}$, and μ_i as the membership functions of $\tilde{\theta}_i$, we randomly generate θ_{ik} from the α-level sets of fuzzy variables $\tilde{\theta}_i, i = 1, 2, \ldots, t$ and $k = 1, 2, \ldots, K$ as $\theta_k = (\theta_{1k}, \theta_{2k}, \ldots, \theta_{tk})$ and $\mu(\theta_k) = \mu_1(\theta_{1k}) \wedge \mu_2(\theta_{2k})\wedge, \ldots, \wedge\mu_t(\theta_{tk})$, where α is a sufficiently small positive number [24,25].

Based on the definition in Appendix A-Eq. (A.3), the expected value of the fuzzy variable is:

$$E\left[\tilde{\theta}\right] = \int_{0}^{+\infty} Cr\left\{\tilde{\theta} \geq r\right\}dr - \int_{-\infty}^{0} Cr\left\{\tilde{\theta} \leq r\right\}dr. \tag{27}$$

Then, provided N is sufficiently large, for any number $r \geq 0$, $Cr\left\{Z(\tilde{\theta}, Q) \geq r\right\}$ can be estimated by:

$$Cr\left\{\tilde{\theta} \geq r\right\} = \frac{1}{2}\left(\operatorname*{Max}_{k=1,2,\ldots,N}\left\{\mu_k \middle| \tilde{\theta} \geq r\right\} + 1 - \operatorname*{Max}_{k=1,2,\ldots,N}\left\{\mu_k \middle| \tilde{\theta} < r\right\}\right). \tag{28}$$

And for any number $r < 0$, $Cr\left\{\tilde{\theta} \leq r\right\}$ can be estimated by:

$$Cr\left\{\tilde{\theta} \leq r\right\} = \frac{1}{2}\left(\operatorname*{Max}_{k=1,2,\ldots,N}\left\{\mu_k \middle| \tilde{\theta} \leq r\right\} + 1 - \operatorname*{Max}_{k=1,2,\ldots,N}\left\{\mu_k \middle| \tilde{\theta} > r\right\}\right). \tag{29}$$

However, the procedure of estimating $E\left[\tilde{\theta}_j\right]$ in Eqs. (28) and (29) is shown in Algorithm.

In fact, by applying the algorithm 1 the fuzzy demand of each product is estimated [24,25].

Algorithm 1. Estimatinge $E[\tilde{\theta}]$
Step 1. Set $E = 0$.
Step 2. Randomly generate θ_{ik} from α-levle sets of fuzzy varibles $\tilde{\theta}_i$, and set $\theta_k = (\theta_{1k}, \theta_{2k}, \ldots, \theta_{nk})$.
Step 3. Set $a = \theta_1 \wedge \theta_2 \wedge \cdots \wedge \theta_K$ and $b = \theta_1 \vee \theta_2 \vee \cdots \vee \theta_K$.
Step 4. Randomly generate r form Uniform $[a, b]$.
Step 5. If $r \geq 0$, *then* $E \leftarrow E + Cr\left\{\tilde{\theta} \geq r\right\}$.
Step 6. If $r < 0$ *then* $E \leftarrow E - Cr\left\{\tilde{\theta} \leq r\right\}$.
Step 7. Repeat the fourth to six steps for N times.
Step 8. $E\left[\tilde{\theta}\right] = a \vee 0 + b \wedge 0 + E \times \frac{b-a}{N}$.

4.2. Rough simulation

In order to estimate the uncertain demands of rough a variable, we employ a simulation technique. Rough simulation plays an important role in rough system [24,25]. If δ_j is a rough vector defined on rough space $(\Lambda, \Delta, A, \pi)$, in order to estimate the expected value $E[\delta]$, rough a simulation approach may be employed. In this approach, by denoting ψ_l by $\psi_l = (\psi_{1j}, \psi_{2j}, \ldots, \psi_{Lj})$ we generate $\underline{\psi}_l = (\underline{\psi}_{1j}, \underline{\psi}_{2j}, \ldots, \underline{\psi}_{Lj})$ from Δ, and $\bar{\psi}_l = (\bar{\psi}_{1j}, \bar{\psi}_{2j}, \ldots, \bar{\psi}_{Lj})$ from Λ according to the measure π [24,25]. So δ_j will be a function of $\delta(\underline{\psi}_l)$ and $\delta(\bar{\psi}_l)$. However, to evaluate $E[\delta_j]$ we need to estimate $\delta(\underline{\psi}_l)$ and $\delta(\bar{\psi}_l)$ [24,25].

Finally the procedure of estimating, $\delta(\underline{\psi}_l)$, $\delta(\bar{\psi}_l)$ and $E[\delta_j]$ in Eq. (26) is shown in Algorithm 2 [24,25].

Algorithm 2. Estimating $E[\delta]$
Step 1. Set $L = 0$.
Step 2. Generate $\underline{\psi}_l$ form Δ according to measure π.
Step 3. Generate $\bar{\psi}_l$ from Λ according to measure π.
Step 4. $L \leftarrow L + \delta(\underline{\psi}) + \delta(\bar{\psi})$.
Step 5. Repeat the two to six steps for N times.
Step 6. $E[\delta] = \frac{L}{2N}$.

Algorithm 2 is applied to estimate the rough demand of each product.

4.3. Harmony search

Meta-heuristic algorithms have been used to solve complex problems for less than a century [26]. Some of the more applied meta-heuristic algorithms are simulating annealing [27], threshold accepting [28], Tabu search [29], genetic algorithms [30], neural networks [31], ant colony optimization [26], fuzzy simulation [32], evolutionary algorithm [33], harmony search (HS) [34–39], and bee colony optimization [21].

HS meta-heuristic algorithm was conceptualized using the musical process of searching for a perfect state of harmony. This state will be obtained by seeking to a find pleasing harmony (a perfect state) as determined by an aesthetic standard, just as the optimization process seeks to find a global solution (a perfect state) based on an objective function [36].

According to the analogy of improvisation of musician group and optimization of an engineering problem, a desirable harmony is considered as global optimum, the aesthetic standard is determined by the objective function, the pitches of instruments are the desired values of the variables, and each practice of musician group is the same as the each iteration improving the solution. The main steps of a HS algorithm are shown by algorithm 3, [35]:

Algorithm 3. HS Algorithm Steps
Step 1. Initialize the algorithm parameters and harmony memory.
Step 2. Improvise a new harmony.
Step 3. Update the harmony memory.
Step 4. Check the stopping criteria.

4.3.1. Initialize the problem and algorithm parameters (Step 1)

Parameters and harmony memory initializations are two parts of the first step of the HS algorithm. The definitions of the HS algorithm's parameters and their values used in this research are shown in Table 1. The *HMS* is the number of simultaneous solution vectors in the harmony memory. The *HMCR* is the probability of choosing each component of the harmony memory. Choosing the value of *HMCR* is very important, using very small *HMCR* will decrease the efficiency of the algorithm and algorithm will work as pure random search. From the literature [35,37–40] it seems that using a small *HMS* is better than using a one and using a large value for the *HMCR* will improve the efficiency.

PAR is the probability of pitch adjustment whose recommended value ranges are from 0.05 to 0.7 [38,39]. The value of N, the number of variables for optimization, is fully dependent on characteristics of the problem. For the proposed HS algorithm in this research N has been chosen to be 6. Finally, *MNI* is the maximum number of iterations that the objective function has evaluated.

The harmony memory which is resulted from the harmony memory initialization, is a two dimensional matrix containing *HMS* rows and $N + 1$ columns. Each row and each column of the harmony memory matrix show a solution vector and each variable respectively. The last column of the proposed matrix is the objective function value of each solution vector. This matrix is generated randomly in a specific range limited by upper and lower bounds determined by the problem at hand. However, because of the constraints described in Section 3, only those solution vectors that satisfy the constraints are included in HM. The considered values of the parameters which used in this research and shown in Table 1 are carried out from the literature [35,37–40].

4.3.2. Improvisation a new harmony

A New Harmony vector $\mathbf{X}' = \left[x'_1, x'_2, \ldots, x'_N\right]$ improvisation is generated based on three rules which are harmony consideration, pitch adjustment and random selection. In memory consideration, the value of each decision variable for the new vector is chosen from any value in the specified HM range. *HMCR*, which varies between 0 and 1, is the rate of choosing one value from the historical values stored in the HM, while $(1 - HMCR)$ is the rate of randomly selecting one value from the possible range of values [38,39].

In the second, pitch adjustment, every component obtained by the memory consideration, is examined to determine whether it should be pitch adjusted or not [38,39]. The value

Table 1: Parameters used in the HS algorithm.

Parameter	Abbreviations	Considered values in this research
Harmony memory size	HMS	(10, 20, 30, and 40)
Memory considering rate	HMCR	(0.85, 0.90, 0.95 and 0.99)
Pitch adjusting rate	PAR	(0.1, 0.3, 0.5 and 0.7)
Number of decision variable	N	(6)
Maximum number of improvisation	MNI	(50, 100, 250 and 500)

Table 2: General data.

Item	P_j	h_j	$\hat{\pi}_j$	f_j	q_{1j}	q_{2j}	q_{3j}	C_{1j}	C_{2j}	C_{3j}	C_{4j}	UL_j	n_j	SL_j
1	7	1	8	3	80	130	180	4	3	2	1	220	10	0.8
2	12	2	6	2	80	130	180	8	5	4	3	220	2	0.8
3	30	4	4	3	80	130	180	20	17	14	12	150	12	0.8
4	30	4	4	4	30	50	70	10	8	6	4	150	6	0.7
5	40	2	8	3	30	50	70	13	11	9	7	100	3	0.7
6	45	5	6	2	30	50	70	15	10	8	6	100	1	0.7

Table 3: Specific data about uncertain demand.

Item	LR fuzzy number			Stochastic	Rough
	$L = R = S(\theta) = \text{Max}\,(0, 1 - \theta)$	$L = R = S(\theta) = \text{Max}\,(0, 1 - \theta)$	$L = R = S(\theta) = e^{-\theta^2}$		
	$\tilde{\theta}_j = (m_j, m_j, \alpha_j, \beta_j)$	$\tilde{\theta}_j = (m_j, n_j, \alpha_j, \beta_j)$	$\tilde{\theta}_j = (m_j, n_j, \alpha_j, \beta_j)$		
	Triangular	Trapezoidal	Exponential	λ_j	δ_j
1	(200, 200, 20, 20)	(195, 205, 15, 15)	(105, 205, 1, 2)	200	[195, 205][180, 220]
2	(225, 225, 15,15)	(220, 230, 10, 10)	(220, 230, 1, 2)	225	[220, 230][210, 240]
3	(115, 115, 15, 15)	(110, 120, 10, 10)	(110, 120, 1, 2)	115	[110, 120][100, 130]
4	(100, 100, 20, 20)	(90, 110, 10, 10)	(90, 110, 1, 2)	100	[95, 105][80, 120]
5	(75, 75, 15, 15)	(70, 80, 10, 10)	(70, 80, 1, 2)	75	[70, 80][60, 90]
6	(30, 30, 10, 10)	(25, 35, 5, 5)	(25, 35, 1, 2)	30	[25, 35][20, 40]

of decision variable is changed by Eq. (30) with probability of *PAR*, and this value is kept without any change with probability $(1 - PAR)$. In Eq. (30) *BW* stands for band width and denotes the amount of change for pitch adjustment, and *rand* is a uniform random number between 0 and 1. In this equation, for each component of the vector the selection for increasing or decreasing are carried out with the same probability [38,39].

$$\mathbf{X}' = \mathbf{X}' \pm (\text{rand})\,(\text{BW}); \quad \text{rand} \sim U\,[0, 1]\,. \tag{30}$$

Finally for random selection which may use the first and second rules, the new value of each decision variable x_i' is randomly chosen within the allowable range of the vector solution $\mathbf{X}^j$.

4.3.3. *Update the harmony memory*

If the new harmony vector is better than the worst one (based on their objective value) in the HM, and no identical harmony vector is stored in the HM, the new harmony is included in the HM and the existing worst one is excluded from the HM [38,39].

The constraint handling part of the algorithm is performed before the HM update. There are five constraints in each of the three proposed models. In fact, constraint handling will check whether all these constraints are satisfied or not. If they are satisfied, then the HM update action occurs [38,39].

4.3.4. *The stopping criterion*

In this step the termination criterion will be checked. The stopping criterion is the maximum number of iterations (*MNI*). In fact the third and fourth steps are repeated until the termination criterion (*MNI*) is satisfied.

Detailed description of the proposed HS algorithm is shown in Appendix B. It should be noted that first by applying Algorithms 1 or 2, the amount of demand for each product is estimated and subsequently the HS algorithm is run to solve the problem [38,39].

In order to demonstrate the proposed HS algorithm and evaluate its performance, in the next section we give five numerical examples.

5. Numerical example

Suppose a dairy product distributer with six products and general data given in Table 2 in which some information is used from [22]. Specific data about three kinds of uncertain demand of each product are shown in Table 3. The first column shows three kinds of fuzzy variable: triangular, trapezoidal and exponential. The second and third columns show the data of stochastic and rough demands.

The total available warehouse space and total available budget in each state are respectively 2000 (m^3) and 6000 ($). In this research all the possible combinations of the parameters of HS method (*HMS*, *HMCR*, *PAR*, *MNI*) that are shown in Table 4 are employed. In fact we have examined 256 combinations of all parameters ($4 \times 4 \times 4 \times 4 = 256$) and used the max (max) criterion. The best combinations of the parameters are presented in Table 5.

Furthermore, the convergence path of the objective function values of the HS algorithm with total discount is shown in Figures 1–3. Figure 1 shows the convergence path of the

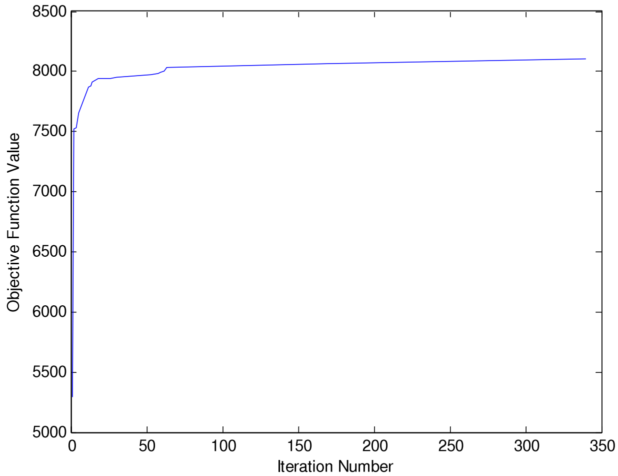

Figure 1: The convergence path of the best result for the triangular fuzzy number.

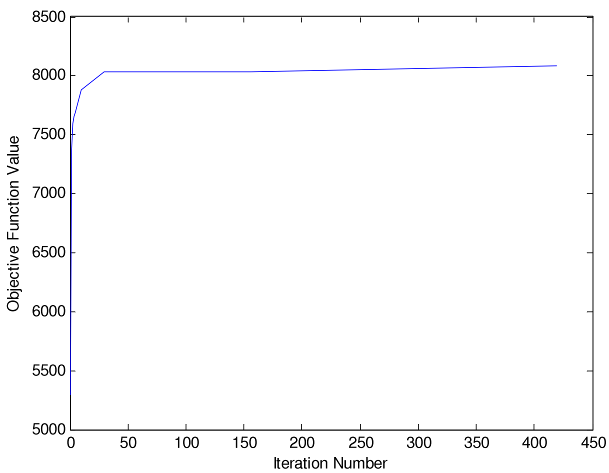

Figure 2: The convergence path of the best result for the stochastic state.

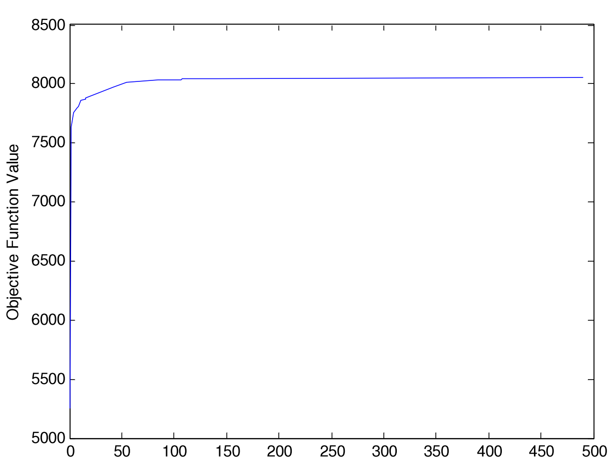

Figure 3: The convergence path of the best result for the rough state.

proposed model with triangular fuzzy variable. It is clear that after less than hundred iterations the algorithm is converged. Figures 2 and 3 show the convergence paths of proposed model with stochastic and rough variables. In these cases it is clear that the algorithm is converged after less than fifty and hundred iterations for stochastic and rough cases respectively.

Table 4: The parameters of the HS algorithm.

HMS	*HMCR*	*PAR*	*MNI*
10	0.85	0.10	50
20	0.90	0.30	100
30	0.95	0.50	250
40	0.99	0.70	500

Table 5: The best combination of the HS parameters.

State	*HMS*	*HMCR*	*PAR*	*MNI*
Triangular fuzzy demand	10	0.95	0.70	500
Trapezoidal fuzzy demand	10	0.95	0.70	500
Exponential fuzzy demand	10	0.95	0.70	250
Stochastic demand	10	0.95	0.70	500
Rough demand	10	0.99	0.70	500

Table 6: The best result for Q_i with incremental discount.

Kind of uncertainty		Product						$Z(Q)$
		1	2	3	4	5	6	
Fuzzy	Triangular	180	216	108	96	84	32	8107
	Trapezoidal	200	218	96	96	78	29	7978
	Exponential	170	204	96	84	90	74	7335
Stochastic		190	218	108	96	72	35	8082
Rough		180	214	108	90	87	27	8050

$K = 15$ and $N = 100$ are considered in the fuzzy simulation and for rough simulation $N = 100$ are considered too. Table 6 shows the best results of all five examples for fuzzy, stochastic and rough demands.

6. Conclusion and recommendations for future research

In this paper, we developed three models for a constrained multi-products single period problem (SPP) with uncertain demands and incremental discount. We assumed that demands may be fuzzy, stochastic and rough variables. We also considered the constraints on service rate, order quantity, warehouse space, and budget. In order to solve the three proposed models, three different kinds of solution algorithm, (1) harmony search, (2) hybrid intelligent based on harmony search and fuzzy simulation and (3) hybrid intelligent based on harmony search and rough simulation are presented. Finally illustrative examples are prepared to show the performance of the developed models and algorithms. The aim of this research was to present models and solution for the situation of uncertainty in demand. If sufficient historical data is available we can use the stochastic model (SSPP). In the case of no historical data and in sufficient knowledge we can use the rough model (RSPP). But if the information at hand is not sufficient to consider the stochastic model we can utilize the fuzzy theory to estimate the demand approximately and therefore use the fuzzy model (FSPP). Using other kinds of Meta-heuristic algorithm to compare against the proposed method and considering uncertain variables for other inventory control parameters such as cost factors are some recommendations for future research. Another interesting research for the future is considering these three models in solving many different problems using simulation to better understand the results and doing an analytical comparison between them.

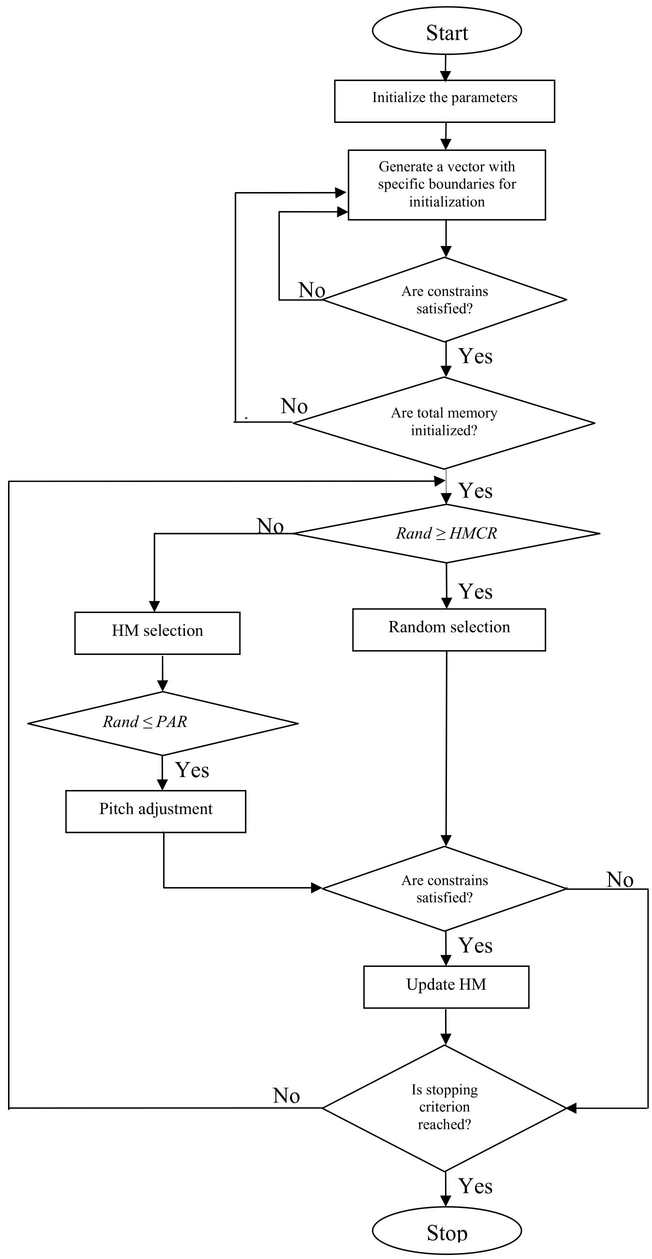

Figure B.1: Flow-chart of the proposed HS algorithm.

Appendix A. Some definitions in fuzzy and rough environments

Let us present some definitions in the fuzzy environment that will be used to model the problem and after that we will present the definitions needed in a rough environment. We adopt the concepts of the credibility, possibility and necessity theory, as well as the credibility of the fuzzy event and the expected value of a fuzzy variable based on [23], Liu [24].

Definition A.1. A Fuzzy number is of *LR*-Type, if there exist reference functions *L* (for the left), *R* (for the right), and scalars $\alpha > 0$, $\beta > 0$ with

$$\mu(\tilde{\xi}) = \begin{cases} 1 & \tilde{\theta} \in [m, n] \\ L\left(\dfrac{m - \tilde{\theta}}{\alpha}\right) & \tilde{\theta} \le m \\ R\left(\dfrac{\tilde{\theta} - n}{\beta}\right) & \tilde{\theta} \ge n. \end{cases} \quad (A.1)$$

And $\tilde{\theta}$ is denoted by $\tilde{\theta} = (m, n, \alpha, \beta)_{L-R}$. The triangular and trapezoidal fuzzy variables are specific kinds of *LR*-Type.

Definition A.2. Let $\tilde{\theta}$ be a fuzzy number with the membership function $\mu(\tilde{\theta})$. Then the possibility, necessity, and credibility measure of the fuzzy event $\tilde{\theta} \geq r$ can be represented, respectively, by:

$$\text{Pos}\left\{\tilde{\theta} > r\right\} = \sup_{\tilde{\xi} \geq r} \mu(\tilde{\theta}) \tag{A.2}$$

$$\text{Nec}\left\{\tilde{\theta} \geq r\right\} = 1 - \sup_{\tilde{\xi} < r} \mu(\tilde{\theta}) \tag{A.3}$$

$$\text{Cr}\left\{\tilde{\theta} \geq r\right\} = \frac{1}{2}\left[\text{Pos}\left\{\tilde{\theta} \geq r\right\} + \text{Nec}\left\{\tilde{\theta} \geq r\right\}\right]. \tag{A.4}$$

Definition A.3. The expected value of fuzzy variable $\tilde{\theta}$ is defined as:

$$E\left[\tilde{\theta}\right] = \int_0^{\infty} Cr\left\{\tilde{\theta} \geq r\right\} dr - \int_{-\infty}^{0} Cr\left\{\tilde{\theta} \leq r\right\} dr. \tag{A.5}$$

For example the expected value of triangular fuzzy variable $\tilde{\theta} = (\theta_1, \theta_2, \theta_3)$ is:

$$E[\tilde{\theta}] = \frac{1}{4}(\theta_1 + 2\theta_2 + \theta_3). \tag{A.6}$$

Definition A.4. Let $\tilde{\theta}$ be a fuzzy variable. Then the optimistic function of α is defined as:

$$\tilde{\xi}_{\sup}(\alpha) = \sup\left[r \left| Cr\left\{\tilde{\theta} \geq r\right\} \geq \alpha\right.\right], \quad \alpha \in (0, 1]. \tag{A.7}$$

Definition A.5. Assume $C_1, C_2, \ldots, C_k$ are real constant and $G_1(\tilde{\theta}), G_2(\tilde{\theta}), \ldots, G_k(\tilde{\theta})$ are functions of the fuzzy variable, then

$$E\left[\sum_{k=1}^{K} C_k G_k(\tilde{\theta})\right] = \sum_{k=1}^{K} C_k E(G_k(\tilde{\theta})). \tag{A.8}$$

Now the definitions in the rough environment needed to modeling the problem are presented based on [23,24,39].

Definition A.6. Let Λ be a nonempty set, A be a $\sigma - Algebra$ of subset of Λ, Δ be an element in A, and π be a nonnegative, real valued, additive set function, then $(\Lambda, \Delta, A, \pi)$ is called rough space [38].

Also, a rough variable is defined as a measurable function from a rough space to a real line.

Definition A.7. A rough variable δ on the rough space $(\Lambda, \Delta, A, \pi)$ is a function from Λ to the real line $\Re$ such that for every boreal set O of $\Re$, we have $\{\vartheta \in \Lambda \,|\delta(\vartheta) \in \text{O}\} \in A$. The lower and upper approximation of the rough variable δ are then respectively defined as follows [39],

$$\underline{\delta} = \{\delta(\vartheta)\,|\vartheta \in \Delta\}, \qquad \bar{\delta} = \{\delta(\vartheta)\,|\vartheta \in \Lambda\}. \tag{A.9}$$

For example a rough variable $([a, b], [c, d])$ with $c \leq a \leq b \leq d$ is a measurable function from a rough space $(\Lambda, \Delta, A, \pi)$ to the real line, where $\Lambda = \{\chi\,|\,c \leq \chi \leq d\}$, $\Delta = \{\chi\,|\,a \leq \chi \leq b\}$ and $\delta(\chi) = \chi$ for all $\chi \in \Lambda$.

Definition A.8. The expected value of an assumed rough variable will be $E[\delta] = \frac{1}{4}[a + b + c + d]$ [39].

Appendix B

Flow-chart of the proposed HS algorithm is shown in Figure B.1.

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